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On off-shell structure of open string sigma model

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Abstract: We analyze several problems related to off-shell structure of open string sigma model by using a combination of derivative expansion and expansion in powers of the fields. According to the sigma model approach to bosonic open string theory, the tachyon effective action $S(T)$ coincides with the renormalized partition function $Z(T)$ of sigma model on a disk, up to a term vanishing on shell. On the other hand, $Z(T)$ is a generating functional of perturbative open string scattering amplitudes. If $S(T) = Z(T)$, then there should be no contribution of exchange diagrams to string amplitudes computed using $S(T)$. We compute the cubic term in the effective action, and show that it vanishes if some but not all external legs are on shell, and, therefore, any exchange diagram involving the cubic term vanishes too. Then, we discuss a problem of turning on non-renormalizable boundary interactions, corresponding to massive string modes. We compute the quadratic term for a symmetric tensor field, and show that despite nonrenormalizability of the model one can consistently remove all divergent terms, and obtain a quadratic action reproducing the on-shell condition for the field. We also briefly discuss fermionic (NS) sigma model, compute the tachyon quadratic term, and show that it reproduces the correct tachyon mass. We note that turning on a massive symmetric tensor field leads to the appearance of a term linear in it, which can be removed by adding a higher-derivative term to the boundary of the disc.

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Contents

1. Introduction

The boundary sigma model approach [1]–[7] (for a review, see [8]) to open string theory has been successfully applied to the study of tachyon condensation [9] in open string theory [10]–[20]. According to the approach, the effective action $S(T, A)$ for the massless vector and tachyon fields in bosonic open string theory is related to the renormalized partition function $Z(T, A)$ of boundary sigma model on the disk as [7]

$$S(T, A) = \left(1 + \beta^T \frac{\partial}{\partial T} + \beta^A \frac{\partial}{\partial A}\right) Z(T, A)\,, \tag{1.1}$$

where β^T and β^A are tachyon and vector fields beta-functions, respectively.

On the other hand, $Z(T, A)$ is a generating functional of perturbative open string scattering amplitudes, and since the beta-functions vanish on shell, then, naively, according to (1.1), a n-point string scattering amplitude would be given just by the corresponding term in $S(T, A)$. This is a puzzle, because one also would expect non-vanishing contribution of exchange diagrams to string amplitudes computed using $S(T, A)$. In the case of the exchange by a massless particle the puzzle was resolved in [3] by noting that the renormalized partition function $Z(T, A)$ does not generate string scattering amplitudes because renormalization of logarithmic infinities corresponds to subtraction of massless poles in the amplitudes. However, this does not explain what happens with the exchange of tachyons. In particular, if one considers the tachyon effective action, i.e. one sets $A = 0$,[1] then one should explain why the tachyon cubic term does not contribute to the 4-point tachyon scattering amplitude through exchange diagrams.

To try to resolve the puzzle, we compute the tachyon cubic term in the effective action $S(T)$, and show that it vanishes if not all external legs are on shell, and, therefore, any exchange diagram involving the cubic terms vanishes too. However, if all external legs are on shell, the cubic term is ill-defined, and to obtain a well-defined string scattering amplitude one has to shift the on-shell mass condition by introducing a soft mass term for the tachyon. We expect that any term in the tachyon effective action possesses the same property, and this gives a resolution of the problem. Being expanded in powers of momenta, the quadratic and cubic terms obtained reproduce the two-derivative tachyon effective action derived in [11]. Let us note that the expression for the same tachyon cubic term obtained in [11] does not match even the cubic term of the tachyon potential derived in [10, 11]. Since no detail of the computation was presented in [11], we cannot say what is a reason of the disagreement.

One may try to generalize the boundary sigma model by turning on non-renormalizable interactions corresponding to massive string modes, i.e. to deal with the boundary string field theory (BSFT) [5, 7]. It is believed that once one includes a massive string mode one has to turn on all the string modes. Although this is so in general, we argue that one can consistently reconstruct the part of the string effective action which depends on all string modes up to some mass level by defining the action as a series in powers of the modes. As an example, we compute quadratic terms for tachyon, vector and symmetric tensor fields, and show that despite the nonrenormalizability of the model one can consistently remove all divergent terms, and obtain a quadratic action reproducing the on-shell conditions for the fields. The same equations of motion were previously derived from the conformal invariance condition of the open string sigma model in [21, 22, 23].

We also briefly discuss the fermionic boundary sigma model. We compute the tachyon quadratic term and show that it exhibits the required zero at $p^2 = 1/2\alpha'$. We then comment on the inclusion of massive modes in the sigma model. We note that turning on a massive symmetric tensor field leads to the appearance of a term linear in it, which can be removed by adding a proper higher-derivative term to the disc boundary. It is unclear how this may influence recent discussions of fermionic BSFT in [24, 25].

The plan of the paper is as follows. In section 2 we compute the tachyon cubic term and show that it does not contribute to scattering amplitudes. In section 3 we discuss massive string modes and compute the quadratic action for tachyon, vector

[1] It can be done consistently because there is no term linear in the vector field in the effective action.

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and symmetric tensor of the second rank. In section 4 we derive the tachyon quadratic action for the fermionic case, and determine a higher-derivative term which can be added to the disc boundary to cancel a term linear in massive symmetric tensor field. In appendix A details of the computation of the tachyon cubic term are presented. We also represent the expression for the cubic term obtained in [11] as a product of Γ-functions, and show that it does not reproduce the correct two-derivative cubic term. In appendix B some useful formulas are collected.

2. Tachyon cubic term and exchange diagrams

The bosonic sigma model with the boundary tachyon interaction is described by the action

$$S = S_\Sigma + S_{\partial\Sigma}\,,$$

where

$$S_\Sigma = \frac{1}{4\pi}\int_\Sigma d^2\,z\;\partial x^\mu \bar{\partial} x_\mu\,,$$

and

$$S_{\partial\Sigma} = \int_0^{2\pi}\frac{d\tau}{2\pi}\,\frac{1}{\epsilon}T(x(\tau))\,.$$

Here Σ is a disc of unit radius, T is the tachyon of the bosonic open string theory, ϵ is a UV cutoff, and $\alpha' = 2$. To find the tachyon effective action, we first have to compute the partition function

$$Z(T) = \langle \mathrm{e}^{-S_{\partial\Sigma}}\rangle\,, \tag{2.1}$$

where the averaging is performed by using the free bulk action S_Σ.

According to the boundary sigma model approach, the partition function has to be computed in the framework of the α'-expansion that is equivalent to the expansion in derivatives of tachyon. It is usually said that the derivative expansion is incompatible with the expansion in powers of tachyon. However, this is so only if one takes tachyon to be near its mass shell.[2] The partition function computation in powers of tachyon is usually done by representing tachyon in the form

$$T(x(\tau)) = \int d^dp\;T(p)\mathrm{e}^{ip\cdot x}\mathrm{e}^{ip\cdot\xi(\tau)}\,. \tag{2.2}$$

Here $p\cdot x \equiv p_\mu x^\mu$, we denote the zero mode of $x(\tau)$ as x, and write

$$x(\tau) = x + \xi(\tau)\,, \qquad \int d\tau\xi(\tau) = 0\,.$$

Then it is easy to see that the derivative expansion just means that one should do all computations in the vicinity of $p = 0$.

[2] Off-shell structure of the sigma model in this case was recently discussed in [26].

The expansion of $Z(T)$ in powers of tachyon up to the third order is given by[3]

$$Z(T,A) = \langle 1\rangle + T + TT + TTT\,, \tag{2.3}$$

where

$$\begin{aligned}
\langle 1\rangle &= \int d^d x\,, \\
T &= -\frac{1}{\epsilon}\int \frac{d\tau}{2\pi}\,\langle\, T(x(\tau))\,\rangle = -\frac{1}{\epsilon}\int d^d x\; T(x)\,, && (2.4)\\
TT &= \frac{1}{2}\frac{1}{\epsilon^2}\int \frac{d\tau}{2\pi}\frac{d\tau'}{2\pi}\,\langle\, T(x(\tau))T(x(\tau'))\,\rangle\,, && (2.5)\\
TTT &= -\frac{1}{6}\frac{1}{\epsilon^3}\int \frac{d\tau_1}{2\pi}\frac{d\tau_2}{2\pi}\frac{d\tau_3}{2\pi}\,\langle\, T(x(\tau_1))T(x(\tau_2))T(x(\tau_3))\,\rangle\,. && (2.6)
\end{aligned}$$

To compute the terms we use (2.2) and the boundary bosonic Green function

$$\begin{aligned}
G_B^{\mu\nu}(\tau) &:= \langle \xi^\mu(\tau)\xi^\nu(0)\rangle = \delta^{\mu\nu}G_B(\tau)\\
&= -\delta^{\mu\nu}\left(2\log\left(\sin^2\left(\frac{\tau}{2}\right)+\epsilon^2\right)+2\log 4\right). && (2.7)
\end{aligned}$$

The details of the computation and the expressions for the terms can be found in the appendix A. Here we only comment on the computation of the quadratic term TT. By using (2.2) and formula (B.1) from appendix B one finds the following behavior of TT at small ϵ

$$TT = \frac{1}{2}(2\pi)^d\int d^dp\; T(p)T(-p)\;\left(\epsilon^{4p^2-2}\frac{\Gamma(\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)}+\frac{1}{\epsilon}\frac{\Gamma(-\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(2p^2)}\right).$$

It is seen from this expression that for small p the first term dominates and exhibits the required zero at the tachyon mass shell $p^2=1/2$. Renormalization of the quadratic term in the partition function is done by renormalizing the bare T as

$$T(p)\;\epsilon^{2p^2-1} = T_R(p)\;\epsilon_R^{2p^2}\,. \tag{2.8}$$

It can be also shown that the same renormalization makes any term T^n in the partition function finite, and, therefore, the exact tachyon beta-function is equal to

$$\beta_T(p) = (2p^2-1)T(p)\,. \tag{2.9}$$

However, if one would interested in TT near the tachyon mass shell, the second term would dominate and give a power divergent term, which would make the tachyon beta-function non linear.

[3] The partition function and the effective action have to be multiplied by the constant which is equal to the value the free theory partition function on the disc (with all boundary fields turned off) and coincides with the D25-brane tension. We omit the multiplier throughout the paper.

The corresponding terms in the effective action can be easily found by using (1.1) and the tachyon beta-function (2.9). Here we list and discuss the results obtained (we omit the subscript R on T in what follows).

$$S_{TT} = -(2\pi)^d \int d^d p \; T(p)T(-p) \; \epsilon_{_R}^{4p^2} \frac{\Gamma(\frac{3}{2} - 2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)} \, . \tag{2.10}$$

Up to the renormalization factor $\epsilon_{_R}^{4p^2}$ the expression coincides with the one obtained in [11]. It is clear that the integrand vanishes at $p^2 = 1/2$, that is the correct tachyon mass. Expanding the tachyon quadratic term in powers of p^2, what is equivalent to the derivative expansion of the effective action, we get the term in the two-derivative approximation

$$S_{TT} = \int d^d x \; \left(-\frac{1}{2} T^2 + (2 - \log(4\epsilon_{_R}^2)) \partial_\mu T \partial^\mu T \right) .$$

The first term $-\frac{1}{2}T^2$ has the correct coefficient that follows from the potential $(1 + T)\mathrm{e}^{-T}$. There are three interesting choices of $\epsilon_{_R}$: *i)* $\epsilon_{_R}^2 = \mathrm{e}/4$ gives the correct tachyon mass already at the two-derivative approximation. *ii)* $\epsilon_{_R} = 1/2$ leads to the two-derivative approximation of the action obtained in [11]. This choice of $\epsilon_{_R}$ seems to be the most convenient to study the tachyon condensation. *iii)* $\epsilon_{_R} = \mathrm{e}/2$. Under this choice of ϵ there is no two-derivative tachyon term in the effective action.

It is worth noting that the integrand in (2.10), in fact, exhibits infinite number of zeroes and poles at $2p^2 = 1 + n$ and $2p^2 = \frac{3}{2} + n$, respectively. This probably indicates that the expansion in powers of T is well defined only in some region of p^2 which includes, however, the tachyon mass shell.

The tachyon cubic term in the effective action (1.1) is given by

$$S_{TTT} = \frac{(2\pi)^d}{3} \int d^d p_i \; \delta\left(\sum_i p_i\right) \; T(p_1)T(p_2)T(p_3) \; \left(\epsilon_{_R}^2\right)^{\sum_i p_i^2} S_T(p_1,p_2,p_3) \, , \tag{2.11}$$

$$S_T(p_1,p_2,p_3) = \frac{\Gamma\left(\frac{1}{2} + 2p_1p_2\right)\Gamma\left(\frac{1}{2} + 2p_1p_3\right)\Gamma\left(\frac{1}{2} + 2p_2p_3\right)\Gamma\left(2 - p_1^2 - p_2^2 - p_3^2\right)}{\pi^{3/2}\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)\Gamma(1-2p_3^2)}$$

$$= \frac{\Gamma(\frac{1}{2} + p_1^2 - p_2^2 - p_3^2)\Gamma(\frac{1}{2} + p_2^2 - p_1^2 - p_3^2)\Gamma(\frac{1}{2} + p_3^2 - p_2^2 - p_1^2)\Gamma(2 - p_1^2 - p_2^2 - p_3^2)}{\pi^{3/2}\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)\Gamma(1-2p_3^2)} ,$$

where we used the momentum conservation $p_1 + p_2 + p_3 = 0$. It is useful to expand the tachyon cubic term in powers of p^2 up to the first order in p^2, and to compare the term in the two-derivative approximation with the one obtained in [11]. This gives a non-trivial check of the cubic term obtained. A simple computation gives

$$S_{TTT} = \int d^d x \; \left(\frac{1}{3} T^3 - 2(1 - \log(4\epsilon_{_R}^2)) T \partial_\mu T \partial^\mu T \right) .$$

Substituting $\epsilon_R = 1/2$ into the expression, we see that it reproduces the cubic term in the two-derivative action obtained in [11].[4]

The cubic term exhibits a rather unusual momentum dependence. One can easily see that if one or two tachyons are on shell, $p_i^2 = 1/2$, then the tachyon cubic term vanishes. This implies that any exchange diagram involving the cubic term vanishes, and, in particular, the 4-point tachyon scattering amplitude should be given just by the corresponding quartic term in the effective action. This is consistent with the fact that the on-shell effective action coincides with the sigma model partition function, that is a generating functional of perturbative tachyon amplitudes. On the other hand if all the three tachyons are on shell, i.e. we are computing the 3-point tachyon amplitude, we have an ill-defined expression $0 \times \infty$. This ambiguity is obviously a manifestation of the Moebius volume infinity [3, 27]. One may regularize the cubic term by adding to the quadratic action a soft mass term m^2T^2. Then it is easy to see that the 3-point amplitude has a finite limit at $m^2 \to 0$. We expect that any n-point term in the tachyon effective action has the same property, and that explains the absence of the contribution of exchange diagrams to perturbative tachyon amplitudes.

3. Massive string modes

In this section we proceed with the study of the boundary sigma model by turning on the vector field and the massive symmetric tensor field of the second rank. We compute the quadratic action for tachyon, vector and massive symmetric tensor fields, and show that it exhibits the required mass-shell conditions for all the fields. Although the inclusion of the massive tensor field leads to nonrenormalizability of the sigma model, the partition function and the action can be made finite by adding proper boundary terms. The action obtained should be regarded as the corresponding part of the quadratic action of the boundary string field theory.

The boundary sigma model with tachyon, vector and symmetric tensor turned on is described by the boundary interaction

$$S_{\partial\Sigma} = \int_0^{2\pi} \frac{d\tau}{2\pi} \left(\frac{1}{\epsilon} T(x(\tau)) - \frac{i}{2} A_\mu(x(\tau))\dot{x}^\mu + \epsilon B_{\mu\nu}\dot{x}^\mu\dot{x}^\nu \right). \tag{3.1}$$

Expanding the partition function (2.1) up to the second order in the fields, we get

$$Z(T, A) = \langle 1 \rangle + T + B + TT + AA + TB + BB\,, \tag{3.2}$$

[4] The tachyon cubic term was also computed in [11], and expressed in terms of the hypergeometric function ${}_3F_2$. We show in appendix A that the expression obtained in [11] does not reproduce the cubic term in the two-derivative action derived in [11].

where T and TT are given by (2.4) and (2.5), and

$$B = -\epsilon \int \frac{d\tau}{2\pi} \langle \; B_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} \; \rangle = -\frac{1}{\epsilon}\int \; d^d x \; B^{\mu}_{\mu}(x)\,, \tag{3.3}$$

$$AA = -\frac{1}{8}\int \frac{d\tau}{2\pi}\frac{d\tau'}{2\pi} \langle \; A_{\mu}(x(\tau))\dot{x}^{\mu}(\tau)A_{\mu'}(x(\tau'))\dot{x}^{\mu'}(\tau') \; \rangle\,, \tag{3.4}$$

$$TB = \int \frac{d\tau}{2\pi}\frac{d\tau'}{2\pi} \langle \; T(x(\tau))B_{\mu\nu}(x(\tau'))\dot{x}^{\mu}(\tau')\dot{x}^{\nu}(\tau') \; \rangle\,, \tag{3.5}$$

$$BB = \frac{1}{2}\epsilon^2 \int \frac{d\tau}{2\pi}\frac{d\tau'}{2\pi} \langle \; B_{\mu\nu}(x(\tau))B_{\mu'\nu'}(x(\tau'))\dot{x}^{\mu}(\tau)\dot{x}^{\nu}(\tau)\dot{x}^{\mu'}(\tau')\dot{x}^{\nu'}(\tau') \; \rangle\,, \tag{3.6}$$

Comparing (2.4) and (3.3), we see that we have to shift T to remove the term linear in B^{μ}_{μ}

$$T(x) \longrightarrow T(x) - B^{\mu}_{\mu}(x)\,. \tag{3.7}$$

Computation of the quadratic terms is done by using formulas from appendix B. The expression for the tachyon quadratic term is given in appendix A. For the vector field quadratic term one gets

$$AA = -\frac{1}{8}(2\pi)^d \int d^d p \; F_{\mu\nu}(p)F_{\mu\nu}(-p)\left[\epsilon^{4p^2}\frac{\Gamma(-\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)} + \right. \\ \left. + \frac{1}{\epsilon}\frac{\Gamma(\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(2+2p^2)}\right]. \tag{3.8}$$

The first term in (3.8) coincides with the one computed in [3] by using the analytical continuation in momenta. We also see that (3.8) contains a power divergent term which can be removed by the tachyon redefinition

$$T(x) \longrightarrow T(x) - \frac{1}{8(2\pi)^d}\int d^d y \; d^d p \; F_{\mu\nu}(x+\frac{y}{2})F_{\mu\nu}(x-\frac{y}{2})\mathrm{e}^{ip\cdot y}\frac{\Gamma(\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(2+2p^2)}\,. \tag{3.9}$$

This tachyon redefinition is a generalization to all orders in derivatives of the one discussed in [8]. After the redefinition the renormalization of the quadratic term in the partition function is done by renormalizing the bare vector field A as

$$A(p) \; \epsilon^{2p^2} = A_R(p) \; \epsilon_R^{2p^2}\,, \tag{3.10}$$

that gives the vector field beta-function

$$\beta_A(p) = 2p^2 A(p)\,. \tag{3.11}$$

Then by using (1.1) and the beta function, one finds the vector field quadratic action[5]

$$S_{AA} = \frac{1}{4}(2\pi)^d \int d^d p \; F_{\mu\nu}(p)F_{\mu\nu}(-p)\epsilon_R^{4p^2}\frac{\Gamma(\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)}\,. \tag{3.12}$$

Expanding the quadratic term in powers of p^2, we obtain the usual $\frac{1}{4}F^2$ term with the conventional coefficient.

[5]Here and in what follows we write quadratic actions for the renormalized fields, and omit the subscript R on T in all the formulas.

The cross term TB is given by

$$TB = -2(2\pi)^d \int d^dp\ T(p)p^\mu p^\nu B_{\mu\nu}(-p)\left[\epsilon^{4p^2}\frac{\Gamma(-\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)}+ \right.$$
$$\left. + \frac{1}{\epsilon}\frac{\Gamma(\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(2+2p^2)}\right] +$$
$$+\frac{1}{\epsilon^2}(2\pi)^d \int d^dp\ T(p)B^\mu_\mu(-p)\ \epsilon^{4p^2}\frac{\Gamma(\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)}\,. \tag{3.13}$$

It can be easily checked that the last term is canceled by a similar term coming from TT after the tachyon shift (3.7). The power divergent term is removed by a redefinition similar to (3.9)

$$T(x) \longrightarrow T(x) + \frac{2}{(2\pi)^d}\int d^dy\ d^dp\ T\left(x+\frac{y}{2}\right)\partial^\mu\partial^\nu B_{\mu\nu}(x-\frac{y}{2})\mathrm{e}^{ip\cdot y}\frac{\Gamma(\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(2+2p^2)}\,.$$

The renormalization of the first term in (3.13) is done by renormalizing the bare $B_{\mu\nu}$ as

$$B_{\mu\nu}(p)\ \epsilon^{2p^2+1} = B^R_{\mu\nu}(p)\ \epsilon_R^{2p^2}\,, \tag{3.14}$$

that gives the beta-function for $B_{\mu\nu}$

$$\beta_B(p) = (2p^2+1)B(p)\,. \tag{3.15}$$

The corresponding term in the action is then found by using (1.1), the tachyon beta-function (2.9) and (3.15)

$$S_{TB} = 4(2\pi)^d \int d^dp\ T(p)p^\mu p^\nu B_{\mu\nu}(-p)\epsilon_R^{4p^2}\frac{\Gamma(\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)}\,. \tag{3.16}$$

The term quadratic in $B_{\mu\nu}$ has the form

$$BB = \frac{1}{2}(2\pi)^d \int d^dp\ \epsilon^{4p^2+2}\left[\frac{1}{\epsilon^4}B^\mu_\mu(p)B^\nu_\nu(-p)\frac{\Gamma(\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)} - \right.$$
$$- \frac{1}{\epsilon^2}p^\mu p^\nu B_{\mu\nu}(p)B^\rho_\rho(-p)\frac{4\ \Gamma(-\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)} +$$
$$+ B_{\mu\nu}(p)B^{\mu\nu}(-p)\frac{2\ \Gamma(-\frac{3}{2}-2p^2)}{\Gamma(1/2)\Gamma(-1-2p^2)} +$$
$$+ p^\mu p^\nu B_{\mu\nu}(p)B^\rho_\rho(-p)\frac{8\ (1+p^2)\Gamma(-\frac{3}{2}-2p^2)}{\Gamma(1/2)\Gamma(-2p^2)} +$$
$$+ \frac{12\ \Gamma(-\frac{3}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)}\left(p^\mu p^\nu B_{\mu\nu}(p)p^\rho p^\sigma B_{\rho\sigma}(-p) - \right.$$
$$\left. - \frac{4}{3}p^2 p^\nu p^\sigma B_{\mu\nu}(p)B_{\mu\sigma}(-p)\right) +$$
$$+ \epsilon^{-4p^2-3}B_{\mu\nu}(p)B_{\rho\sigma}(-p)\tilde{K}^{\mu\nu\rho\sigma}(p) +$$
$$\left. + \epsilon^{-4p^2-1}B_{\mu\nu}(p)B_{\rho\sigma}(-p)K^{\mu\nu\rho\sigma}(p)\right]\,. \tag{3.17}$$

Here $K^{\mu\nu\rho\sigma}(p)$ and $\tilde{K}^{\mu\nu\rho\sigma}(p)$ are tensors which do not depend on ϵ. Explicit expressions for the last two terms are given in appendix B.

It is not difficult to check that the first two terms in (3.17) are canceled by the corresponding terms coming from TT and TB after the tachyon shift (3.7). The next three terms are finite in terms of the renormalized fields $B^R_{\mu\nu}$, eq. (3.14). The power divergent term (with $\tilde{K}$) is again removed by a tachyon shift of the form (3.9). The last term reflects the nonrenormalizability of the model. Being expressed in terms of the renormalized fields, it takes the form

$$\frac{1}{2}(2\pi)^d \int d^dp\, \frac{1}{\epsilon}\left(\frac{\epsilon}{\epsilon_{_R}}\right)^{-4p^2} B^R_{\mu\nu}(p)B^R_{\rho\sigma}(-p)K^{\mu\nu\rho\sigma}(p)\,, \tag{3.18}$$

which shows explicitly that it diverges as $\epsilon \to 0$. Thus, to have a finite partition function one has to cancel the term. It can be done by adding to the boundary interaction (3.1) some higher-derivative term. The requirement that the additional boundary term has a minimal number of derivatives fixes the form of the term to be

$$S'_{\partial\Sigma} = \int_0^{2\pi} \frac{d\tau}{2\pi}\, \epsilon^3 K(B(x(\tau)))\left(\dot{x}^\mu\dot{x}_\mu\dot{x}^\nu\dot{x}_\nu - \frac{2(d+2)}{3}\ddot{x}^\mu\ddot{x}_\mu\right), \tag{3.19}$$

where $K(B(x(\tau)))$ is the following functional of bare $B_{\mu\nu}$

$$\begin{aligned} K(B(x(\tau))) = -\frac{3}{2d(d+2)}\frac{1}{2(2\pi)^d}\int d^dy\, d^dp\, B_{\mu\nu}\left(x(\tau)+\frac{y}{2}\right) B_{\rho\sigma}\left(x(\tau)-\frac{y}{2}\right) \times \\ \times\, \mathrm{e}^{ip\cdot y}K^{\mu\nu\rho\sigma}(p)\,, \end{aligned} \tag{3.20}$$

and the constant $2(d+2)/3$ in (3.19) ensures the absence of terms diverging as $1/\epsilon$. Thus, despite the nonrenormalizability of the model, there is a well defined and consistent way of removing divergent terms.

Now, having the finite partition function, and using the tensor field beta-function (3.15), we find the quadratic action for tachyon and $B_{\mu\nu}$

$$\begin{aligned} S = (2\pi)^d \int d^dp\, \epsilon_{_R}^{4p^2} \Bigg[& -T(p)T(-p)\frac{\Gamma(\frac{3}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)} + \\ & + 4T(p)p^\mu p^\nu B_{\mu\nu}(-p)\frac{\Gamma(\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)} - \\ & - 2B_{\mu\nu}(p)B^{\mu\nu}(-p)\frac{\Gamma(-\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(-1-2p^2)} - \\ & - 8p^\mu p^\nu B_{\mu\nu}(p)B^\rho_\rho(-p)\frac{(1+p^2)\Gamma(-\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(-2p^2)} - \\ & - 12\frac{\Gamma(-\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)}\bigg(p^\mu p^\nu B_{\mu\nu}(p)p^\rho p^\sigma B_{\rho\sigma}(-p) - \\ & - \frac{4}{3}p^2p^\nu p^\sigma B_{\mu\nu}(p)B_{\mu\sigma}(-p)\bigg)\Bigg]. \end{aligned} \tag{3.21}$$

Let us recall that all the fields in (3.21) are renormalized. We can remove the cross term TB by shifting T

$$T(p) \longrightarrow T(p) + \frac{4}{1-4p^2} p^\mu p^\nu B_{\mu\nu}(p)\,. \tag{3.22}$$

Then we have a quadratic action for $B_{\mu\nu}$ which can be easily shown to lead to the usual on-shell conditions

$$2p^2 = -1\,, \qquad B^\mu_\mu = 0\,, \qquad p^\nu B_{\mu\nu}(p) = 0\,.$$

The mass shell condition $2p^2 = -1$ immediately follows from the term

$$B_{\mu\nu}(p) B^{\mu\nu}(-p) \frac{\Gamma(-\frac{1}{2} - 2p^2)}{\Gamma(1/2)\Gamma(-1-2p^2)}$$

in the quadratic action. The action obtained should be regarded as the quadratic part of the BSFT action for the massive symmetric tensor field.

Some comments are in order. It seems that one can make any higher-order term in the sigma model partition function finite by using the same procedure: one first removes power divergent terms by a tachyon redefinition, then one assumes that at any order in the fields their renormalization is given by the formulas (2.8), (3.10) and (3.14), and expresses the partition function in terms of the renormalized fields, and, finally, one cancels all remaining singular terms by adding proper higher-derivative boundary terms. Moreover, it seems possible to turn on all massive modes up to some mass level k, and to perform all computations in the same way as was done for $B_{\mu\nu}$. It will be necessary to shift lower level massive modes to remove power singularities coming from more massive modes, and to assume that a massive mode $B_{(k)}$ of level k is renormalized as

$$B_{(k)}(p)\ \epsilon^{2p^2+k} = B^R_{(k)}(p)\ \epsilon_R^{2p^2}\,. \tag{3.23}$$

Although it is unclear if the level truncated BSFT action constructed this way is unique up to fields redefinitions, we should note that the quadratic action (3.21) was derived unambiguously. One may ask what is the meaning of the level truncated BSFT action. There are two possible answers. The first one is that it is just a level truncated action, and the only requirement is that its quadratic part reproduces the usual on-shell conditions for massive modes. The second, much more optimistic, answer is that it is in fact an effective action for all string modes up to level k, obtained by integrating out all higher level massive modes, i.e. the following equality holds

$$S_{\text{eff}}(T, A, B_{(2)}, \ldots, B_{(k)}) = S_{\text{bsft}}(T, A, B_{(2)}, \ldots, B_{(k)}, 0, \ldots)\,. \tag{3.24}$$

It is clear that (3.24) generalizes the usual assumption that the effective action for the tachyon and vector fields coincides with the BSFT action with all massive modes turned off. If eq. (3.24) would hold then there would be no term linear in any massive mode in the BSFT action, i.e. by a field redefinition the part of the BSFT action depending on massive modes could be brought to the following form

$$S_{\mathrm{bsft}}(B_{(n)}) = c_k^{(2)}(T,A)B_{(k)}^2 + c_k^{(3)}(T,A)B_{(k)}^3 + c_k^{(4)}(T,A)B_{(k)}^4 + c_{kl}(T,A)B_{(k)}^2 B_{(l)}^2 + \\ + \cdots . \tag{3.25}$$

Equation (3.24) imposes strong constraints on a possible form of the BSFT action obtained from the boundary sigma model with all string modes turned on, because before a fields redefinition the BSFT action contains infinite number of terms linear in any massive mode, for example, we have a term $T\partial^2 B$ in eq. (3.21), and, in general, there are terms of the form $T^n\partial^2 B$. It does not seem possible to prove the existence of a fields redefinition reducing the BSFT action to the form (3.25), and, therefore, to justify (3.24). Let us note, however, that the fact that computing string amplitudes from the sigma model partition function does not require turning on all string modes may serve as an argument in favor of (3.24).

4. Fermionic sigma model

In this section we first compute the quadratic term in the effective action for tachyon on an unstable D9-brane in type-II string theory, and show that it reproduces the correct tachyon mass. Then, we turn on a massive symmetric tensor $B_{\mu\nu}$, and show that there is a term linear in B^μ_μ, which remains finite after renormalization.

4.1 Tachyon quadratic action

Tachyon on an unstable D9-brane in type-II string theory is described by sigma model with the action [28, 15]

$$S = S_\Sigma + S_{\partial\Sigma}\,,$$

where

$$S_\Sigma = \frac{1}{4\pi}\int d^2\,z\left(\partial x^\mu\bar\partial x_\mu + \psi^\mu\bar\partial\psi_\mu + \tilde\psi^\mu\partial\tilde\psi_\mu\right)$$

and

$$S_{\partial\Sigma} = \frac{1}{4}\int\frac{d\tau}{2\pi}\left(\frac{1}{\epsilon}T^2 + \frac{1}{\epsilon}\left(\partial_\mu T\psi^\mu\right)\partial_\tau^{-1}\left(\partial_\nu T\psi^\nu\right)\right).$$

The tachyon effective action just coincides with the sigma model partition function

$$S(T) = Z(T) = \langle \mathrm{e}^{-S_{\partial\Sigma}}\rangle\,, \tag{4.1}$$

and, therefore, its quadratic part is given by

$$S_{TT} = \left\langle -\frac{1}{4\epsilon}\int\frac{d\tau}{2\pi}\left(T(x(\tau))^2 + \left(\partial_\mu T(x(\tau))\psi^\mu\right)\partial_\tau^{-1}\left(\partial_\nu T(x(\tau))\psi^\nu\right)\right)\right\rangle \tag{4.2}$$

$$= -\frac{1}{4\epsilon}\int d^dx\; T^2 - \frac{1}{8\epsilon}\int\frac{d\tau}{2\pi}d\tau'\;\langle\;\partial_\mu T(x(\tau))\psi^\mu(\tau)\varepsilon(\tau-\tau')\partial_\nu T(x(\tau'))\psi^\nu(\tau')\;\rangle\,,$$

where $\varepsilon(\tau) = +1$ for $\tau > 0$ and $\varepsilon(\tau) = -1$ for $\tau < 0$.

To compute the quadratic term one needs the boundary fermionic Green function

$$G_F^{\mu\nu}(\tau) := \langle\psi^\mu(\tau)\psi^\nu(0)\rangle = \delta^{\mu\nu}G_F(\tau) = -\delta^{\mu\nu}\frac{2\sin(\tau/2)}{\sin^2(\tau/2)+\epsilon^2}\,, \tag{4.3}$$

and the bosonic Green function (2.7). By using the Green functions and the momentum representation (2.2), one finds

$$S_{TT}^{(1)} = -\int d^dx\frac{1}{4\epsilon}\int\frac{d\tau}{2\pi}\,\langle\, T(x(\tau))T(x(\tau))\rangle = -\frac{1}{4\epsilon}(2\pi)^d\int d^dpT(p)T(-p) \tag{4.4}$$

and

$$\begin{aligned} S_{TT}^{(2)} &= -\int d^dx\frac{1}{8\epsilon}\int\frac{d\tau}{2\pi}d\tau'\,\langle\,\partial_\mu T(x(\tau))\psi^\mu(\tau)\varepsilon(\tau-\tau')\partial_\nu T(x(\tau'))\psi^\nu(\tau')\,\rangle \\ &= \frac{1}{4}(2\pi)^{d+1}\int d^dpT(p)T(-p)\;p^2\;\epsilon^{4p^2-1}\int_0^{2\pi}\frac{d\tau}{2\pi}\sin\left(\frac{\tau}{2}\right)\left[\sin^2\left(\frac{\tau}{2}\right)+\epsilon^2\right]^{-2p^2-1}. \end{aligned} \tag{4.5}$$

Computing the integral over τ, we obtain in the limit $\epsilon\to 0$

$$S_{TT}^{(2)} = \frac{1}{4}(2\pi)^d\int d^dp\;T(p)T(-p)\left(\frac{1}{\epsilon}-\epsilon^{4p^2-1}\frac{\Gamma(1/2)\Gamma(1-2p^2)}{\Gamma(\frac{1}{2}-2p^2)}\right). \tag{4.6}$$

Combining (4.4) and (4.6), we derive the quadratic term in the partition function

$$S_{TT} = Z_{TT}^{(1)}+Z_{TT}^{(2)} = -\frac{1}{4}(2\pi)^d\int d^dp\;T(p)T(-p)\;\epsilon^{4p^2-1}\frac{\Gamma(1/2)\Gamma(1-2p^2)}{\Gamma(\frac{1}{2}-2p^2)}\,. \tag{4.7}$$

Thus, we see that the power divergent terms cancel each other, and the quadratic term can be made finite by the tachyon renormalization

$$T(p)\;\epsilon^{2p^2-\frac{1}{2}} = T_R(p)\;\epsilon_{_R}^{2p^2}\,. \tag{4.8}$$

Then, the integrand exhibits zero at $p^2 = 1/4$, which is the correct tachyon mass in open superstring theory. We also see that if one would use the definition (1.1) for the tachyon action and the tachyon beta-function, $\beta_T(p) = 2p^2-\frac{1}{2}$, one would get the term

$$\tilde{S}_{TT} = -(2\pi)^d\int d^dp\;T(p)T(-p)\;\epsilon^{4p^2-1}p^2\frac{\Gamma(1/2)\Gamma(1-2p^2)}{\Gamma(\frac{1}{2}-2p^2)}\,, \tag{4.9}$$

which exhibits an additional zero at $p^2 = 0$. Expanding the quadratic action in powers of p we get

$$S_{TT} = \frac{1}{4}(2\pi)^d\int d^dpT(p)T(-p)\;\left(-1+(8\log 2-4\log 2\epsilon_{_R})p^2\right).$$

If we choose $\epsilon_{_R} = 1/2$ in this expression, we reproduce the quadratic term that follows from the action found in [15]. If we choose $\epsilon_{_R} = 2/e$ we reproduce the 2-derivative action with the correct tachyon mass.

4.2 Massive symmetric tensor

In this subsection we set tachyon $T = 0$, and only consider a massive symmetric tensor field $B_{\mu\nu}$. The boundary interaction describing the field is given by

$$S_{\partial\Sigma} = \frac{1}{4}\int \frac{d\tau}{2\pi}\; d\theta\; \epsilon B_{\mu\nu}(X) D^2 X^\mu D X^\nu \,. \tag{4.10}$$

Here

$$X^\mu = x^\mu(\tau) + \theta\psi^\mu(\tau)\,, \qquad D = \partial_\theta + \theta\partial_\tau$$

are matter superfields and a supercovariant derivative, respectively. Integrating over θ, we get the component form of the boundary term

$$S_{\partial\Sigma} = \frac{1}{4}\int \frac{d\tau}{2\pi}\; \left(\epsilon B_{\mu\nu}\left(\dot{x}^\mu\dot{x}^\nu + \dot{\psi}^\mu\psi^\nu\right) + \epsilon\partial_\rho B_{\mu\nu}\dot{x}^\mu\psi^\rho\psi^\nu\right). \tag{4.11}$$

To compute correlators in this model we need regularized bosonic and fermionic boundary Green functions. We cannot use the functions (2.7) and (4.3) because they do not preserve 1-d supersymmetry. They violate supersymmetry only at order $o(\epsilon)$, and by this reason we could use them to compute the tachyon quadratic action, but once we turn on a non-renormalizable interaction we are to use Green functions exactly preserving supersymmetry.[6] A possible choice is

$$G_B^\epsilon(\tau) = 4\sum_{k=1}^{\infty} \mathrm{e}^{-2k\epsilon}\frac{\cos k\tau}{k} = -2\left(\log\left(\sin^2\left(\frac{\tau}{2}\right) + \sinh^2(\epsilon)\right) + \log 4 - 2\epsilon\right), \tag{4.12}$$

and

$$G_F^\epsilon(\tau) = -4\sum_{r=1/2}^{\infty} \mathrm{e}^{-2r\epsilon}\sin r\tau = -\frac{2\cosh(\epsilon)\sin(\tau/2)}{\sin^2(\tau/2) + \sinh^2(\epsilon)}\,. \tag{4.13}$$

By using the Green functions one can easily compute terms linear in B_μ^μ in the effective action (4.1)

$$S^{(1)}(B) = \frac{1}{8}\int d^{10}x\; \epsilon B_\mu^\mu\,. \tag{4.14}$$

Renormalization of $B_{\mu\nu}$ (see, (3.14)) just absorbs ϵ, and one gets a finite linear term. Contrary to the bosonic case, this linear term cannot be removed by a tachyon redefinition because there is no term linear in tachyon in the fermionic case. On the other hand, this term would violate the usual on-shell conditions for $B_{\mu\nu}$. Thus we have to cancel it somehow. A possible way is to add a proper higher-derivative term

[6] Strictly speaking, the supersymmetry is spontaneously broken by the antiperiodicity of the boundary fermions [4]. What we mean by saying that the regularization is supersymmetric is that it would preserve the supersymmetry if the fermions were periodic.

to the disc boundary. It is not difficult to show that the simplest choice is

$$\begin{aligned}
S'_{\partial\Sigma} &= \frac{1}{4}\int \frac{d\tau}{2\pi}\, d\theta \left(-\frac{1}{120}\epsilon^3 B^\mu_\mu(X) D^2X^\nu D^2X_\nu D^2X^\rho DX_\rho\right) \\
&= -\frac{\epsilon^3}{480}\int \frac{d\tau}{2\pi}\ \left(B^\mu_\mu\left(\dot{x}^\nu\dot{x}_\nu\dot{x}^\rho\dot{x}_\rho + \dot{x}^\nu\dot{x}_\nu\dot{\psi}^\rho\psi_\rho + 2\dot{x}^\nu\dot{x}_\rho\dot{\psi}^\nu\psi^\rho\right) + \right. \\
&\qquad\qquad \left. + \partial_\sigma B^\mu_\mu \dot{x}^\nu\dot{x}_\nu\dot{x}^\rho\psi^\sigma\psi_\rho\right). \qquad (4.15)
\end{aligned}$$

The coefficient in front of the term was fixed by using

$$\langle \dot{x}^\nu\dot{x}_\nu\dot{x}^\rho\dot{x}_\rho + \dot{x}^\nu\dot{x}_\nu\dot{\psi}^\rho\psi^\rho + 2\dot{x}^\nu\dot{x}_\rho\dot{\psi}^\nu\psi^\rho\rangle = 30\left(-\frac{2}{\epsilon^2}+\frac{7}{6}+o(\epsilon)\right).$$

Although one can cancel the linear term by adding to the disc boundary such a higher-derivative term, the procedure does not look completely satisfactory. The choice of a higher-derivative term is not unique, and different choices would lead to different off-shell actions for $B_{\mu\nu}$, and it is unclear how to argue that they are equivalent.

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A. Tachyon cubic term

Here we list the results obtained for the quadratic and cubic terms in the partition function

$$\begin{aligned}
TT &= \frac{1}{2\epsilon^2}(2\pi)^d \int d^dp\ T(p)T(-p)\ \epsilon^{4p^2}\frac{\Gamma(\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)}, \\
TTT &= -\frac{(2\pi)^d}{6\epsilon^3}\int d^dp_i\delta\left(\sum_i p_i\right) T(p_1)T(p_2)T(p_3)\ \left(\epsilon^2\right)^{\sum_i p_i^2} K_T(p_1,p_2,p_3), \\
K_T(p_1,p_2,p_3) &= \frac{\Gamma(\frac{1}{2}+2p_1p_2)\Gamma(\frac{1}{2}+2p_1p_3)\Gamma(\frac{1}{2}+2p_2p_3)\Gamma(1-p_1^2-p_2^2-p_3^2)}{\pi^{3/2}\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)\Gamma(1-2p_3^2)} \qquad (A.1) \\
&= \frac{\Gamma(\frac{1}{2}+p_1^2-p_2^2-p_3^2)\Gamma(\frac{1}{2}+p_2^2-p_1^2-p_3^2)\Gamma(\frac{1}{2}+p_3^2-p_2^2-p_1^2)\Gamma(1-p_1^2-p_2^2-p_3^2)}{\pi^{3/2}\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)\Gamma(1-2p_3^2)}.
\end{aligned}$$

The derivation of the quadratic terms is straightforward, and will be omitted here.

To compute the cubic term TTT we first note, by using the momentum representation (2.2) and the bosonic Green function (2.7), that the kernel K_T is given by

$$
\begin{aligned}
K_T(p_1,p_2,p_3) &= \left(\epsilon^2\right)^{-\sum_i p_i^2} \left\langle \mathrm{e}^{ip_1\xi(\tau_1)}\mathrm{e}^{ip_2\xi(\tau_2)}\mathrm{e}^{ip_3\xi(\tau_3)}\right\rangle \\
&= \int \frac{d\tau_1}{2\pi}\frac{d\tau_2}{2\pi}\frac{d\tau_3}{2\pi} \left[\sin^2\left(\frac{\tau_{12}}{2}\right)+\epsilon^2\right]^{2p_1p_2} \left[\sin^2\left(\frac{\tau_{13}}{2}\right)+\epsilon^2\right]^{2p_1p_3} \times \\
&\qquad \times \left[\sin^2\left(\frac{\tau_{23}}{2}\right)+\epsilon^2\right]^{2p_2p_3}, \qquad \text{(A.2)}
\end{aligned}
$$

where $\tau_{ij} = \tau_i - \tau_j$.

We are interested in the limit $\epsilon \to 0$ of the integral. Since the limit exists for small values of p_i, all we have to do is to compute the integral for $\epsilon = 0$

$$
K_T(p_1,p_2,p_3) = \int \frac{d\tau_1}{2\pi}\frac{d\tau_2}{2\pi} \left[\sin^2\left(\frac{\tau_{12}}{2}\right)\right]^{2p_1p_2} \left[\sin^2\left(\frac{\tau_1}{2}\right)\right]^{2p_1p_3} \left[\sin^2\left(\frac{\tau_2}{2}\right)\right]^{2p_2p_3}.
$$

To compute the integral we first make a change

$$
\mathrm{e}^{i\tau_1} = \frac{x_1 - i}{x_1 + i}, \qquad \mathrm{e}^{i\tau_2} = \frac{x_2 - i}{x_2 + i},
$$

i.e. we transform the circles into straight lines. Then K_T acquires the form

$$
\begin{aligned}
K_T &= \frac{1}{\pi^2}\int_{-\infty}^{\infty} \frac{dx_1 dx_2}{(1+x_1^2)(1+x_2^2)} \left(\frac{(x_1-x_2)^2}{(1+x_1^2)(1+x_2^2)}\right)^{2p_1p_2} \left(\frac{1}{1+x_1^2}\right)^{2p_1p_3} \times \\
&\qquad \times \left(\frac{1}{1+x_2^2}\right)^{2p_2p_3} \\
&= \frac{1}{\pi^2}\int_{-\infty}^{\infty} dx_1 dx_2 \left((x_1-x_2)^2\right)^{2p_1p_2} \left(1+x_1^2\right)^{2p_1^2-1} \left(1+x_2^2\right)^{2p_2^2-1}. \qquad \text{(A.3)}
\end{aligned}
$$

By using the formula

$$
z^{-a} = \frac{1}{\Gamma(a)}\int_0^{\infty} dt\ t^{a-1}\mathrm{e}^{-tz} \qquad \text{(A.4)}
$$

we rewrite K_T in the form

$$
\begin{aligned}
K_T &= \frac{1}{\pi^2\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)} \int_0^{\infty} dt_1 dt_2 t_1^{-2p_1^2} t_2^{-2p_2^2} \\
&\qquad \times \int_{-\infty}^{\infty} dx_1 dx_2 \left((x_1-x_2)^2\right)^{2p_1p_2} \mathrm{e}^{-t_1(1+x_1^2)-t_2(1+x_2^2)}.
\end{aligned}
$$

Integrating over x_1, x_2, one gets

$$
\begin{aligned}
K_T = \frac{\Gamma(\frac{1}{2}+2p_1p_2)}{\pi^{3/2}\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)} \int_0^{\infty} & dt_1 dt_2 t_1^{-2p_1^2-2p_1p_2-\frac{1}{2}} t_2^{-2p_2^2-2p_1p_2-\frac{1}{2}} (t_1+t_2)^{2p_1p_2} \times \\
& \times\ \mathrm{e}^{-t_1-t_2}
\end{aligned}
$$

By using again (A.4), we obtain

$$K_T = \frac{\Gamma(\frac{1}{2}+2p_1p_2)}{\pi^{3/2}\Gamma(-2p_1p_2)\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)}\int_0^\infty dx x^{-2p_1p_2-1} dt_1 dt_2 \mathrm{e}^{-x(t_1+t_2)} \times$$
$$\times\ t_1^{-2p_1^2-2p_1p_2-\frac{1}{2}} t_2^{-2p_2^2-2p_1p_2-\frac{1}{2}} \mathrm{e}^{-t_1-t_2}\,.$$

Integrating over t_1, t_2, we finally arrive at

$$\begin{aligned}
K_T &= \frac{\Gamma(\frac{1}{2}+2p_1p_2)\Gamma(\frac{1}{2}-2p_1^2-2p_1p_2)\Gamma(\frac{1}{2}-2p_2^2-2p_1p_2)}{\pi^{3/2}\Gamma(-2p_1p_2)\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)} \times \\
&\quad \times \int_0^\infty dx \frac{(1+x)^{2p_1^2+4p_1p_2+2p_2^2-1}}{x^{2p_1p_2+1}} \\
&= \frac{\Gamma(\frac{1}{2}+2p_1p_2)\Gamma(\frac{1}{2}-2p_1^2-2p_1p_2))\Gamma(\frac{1}{2}-2p_2^2-2p_1p_2)\Gamma(1+2p_1p_2-2p_3^2)}{\pi^{3/2}\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)\Gamma(1-2p_3^2)} \\
&= \frac{\Gamma(\frac{1}{2}+p_1^2-p_2^2-p_3^2)\Gamma(\frac{1}{2}+p_2^2-p_1^2-p_3^2)\Gamma(\frac{1}{2}+p_3^2-p_2^2-p_1^2)\Gamma(1-p_1^2-p_2^2-p_3^2)}{\pi^{3/2}\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)\Gamma(1-2p_3^2)}\,,
\end{aligned}$$

where we used the momentum conservation

$$p_1+p_2+p_3=0\,.$$

The computation of TTT was obviously equivalent to the one done by the analytical continuation in momenta. One may ask if one can use the analytical continuation to compute the cubic term AAT involving the open string vector field. The result of the computation is given by

$$\begin{aligned}
AAT &= -\frac{(2\pi)^d}{2\epsilon}\int d^dp_i\delta\left(\sum_i p_i\right) A_{\mu_1}(p_1)A_{\mu_2}(p_2)T(p_3)\ \left(\epsilon^2\right)^{\sum_i p_i^2} K_A(p_1,p_2,p_3)\times \\
&\quad \times \left(p_1^2p_2^2\left(\delta^{\mu_1\mu_2}-4p_2^{\mu_1}p_1^{\mu_2}\right)+(1-4p_1p_2)\left(p_1p_2p_1^{\mu_1}p_2^{\mu_2}-p_2^2p_1^{\mu_1}p_1^{\mu_2}-\right.\right. \\
&\quad \left.\left. -p_1^2p_2^{\mu_1}p_2^{\mu_2}\right)\right) \\
&= -\frac{(2\pi)^d}{2\epsilon}\int d^dp_i\delta\left(\sum_i p_i\right) T(p_3)\ \left(\frac{\epsilon^2}{4}\right)^{\sum_i p_i^2} K_A(p_1,p_2,p_3)\times \\
&\quad \times \left(\frac{1}{2}p_1p_2F_{\mu\nu}(p_1)F^{\mu\nu}(p_2)-F_{\mu\nu}(p_1)F^{\mu\rho}(p_2)\left(p_1^\nu p_2^\rho+p_2^\nu p_1^\rho\right)-\right. \\
&\quad \left. -4F_{\mu\nu}(p_1)F_{\rho\sigma}(p_2)p_1^\mu p_2^\nu p_1^\rho p_2^\sigma\right)\,, \qquad (A.5)
\end{aligned}$$

$$\begin{aligned}
K_A(p_1,p_2,p_3) &= \frac{\Gamma(\frac{1}{2}+2p_1p_3)\Gamma(\frac{1}{2}+2p_2p_3)\Gamma(-\frac{1}{2}+2p_1p_2)\Gamma(-p_1^2-p_2^2-p_3^2)}{\pi^{3/2}\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)\Gamma(1-2p_3^2)} \qquad (A.6) \\
&= \frac{\Gamma(\frac{1}{2}+p_1^2-p_2^2-p_3^2)\Gamma(\frac{1}{2}+p_2^2-p_1^2-p_3^2)\Gamma(-\frac{1}{2}+p_3^2-p_2^2-p_1^2)\Gamma(-p_1^2-p_2^2-p_3^2)}{\pi^{3/2}\Gamma(1-2p_1^2)\Gamma(1-2p_2^2)\Gamma(1-2p_3^2)}\,,
\end{aligned}$$

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We see that the kernel K_A has a pole at $p_i = 0$, and, therefore, does not admit the expansion in powers of momenta. That probably means that one cannot use the analytical continuation to compute the cubic term AAT. Note, however, that the term S_{AAT} in the effective action (1.1) computed by using (A.5) does admit the expansion in powers of momenta.

Now we discuss the expression for the tachyon cubic term obtained in [11]. It has the form (see, [11, (A.8)])

$$S^{(3)} = -\frac{(2\pi)^d}{6}\int d^d p_i\ \delta\left(\sum_i p_i\right)\ T(p_1)T(p_2)T(p_3)\ \ (\epsilon_{_R}^2)^{\sum_i p_i^2} K^{(3)} \times \times\ (p_1, p_2, p_3)\,, \tag{A.7}$$

$$K^{(3)}(p_1, p_2, p_3) = 4^{1-a-b-c}(1-2p_2^2)\frac{\Gamma(1+2a)\Gamma(1+2b)}{\Gamma^2(1+a-c)\Gamma^2(1+b-c)}\cdot \cdot\ {}_3F_2(-2c, -a-c, -c-b; 1+a-c, 1+b-c; 1)\,. \tag{A.8}$$

where

$$a = p_1^2 + p_2^2 - p_3^2 + 1\,, \qquad b = p_1^2 + p_3^2 - p_2^2\,, \qquad c = p_2^2 + p_3^2 - p_1^2\,.$$

We also included the renormalization factor to simplify the comparison with the two-derivative tachyon effective action obtained in [11]. By using formula [29, (4.4.5)], we can rewrite $K^{(3)}$ as follows

$$K^{(3)}(p_1, p_2, p_3) = \frac{4^{1-a-b-c}(1-2p_2^2)\Gamma(1+2a)\Gamma(1+2b)\Gamma(1-c)\Gamma(1+a+b+c)}{\Gamma(1+a-c)\Gamma(1+b-c)\Gamma(1-2c)\Gamma(1+a)\Gamma(1+b)\Gamma(1+a+b)}\,.$$

Expanding (A.7) in powers of momenta, we get

$$S^{(3)} = -\int d^d x\ \left(\frac{1}{3}T^3 - \frac{2}{3}\left(-1 + 4\gamma + 3\log\left(\frac{4}{\epsilon_{_R}^2}\right)\right) T\partial_\mu T\partial^\mu T\right),$$

where γ is the Euler constant. It is clear that the sign in front of T^3 is wrong. Substituting $\epsilon_{_R} = 1/2^7$ into the expression, we also see that it does not reproduce the two-derivative cubic term in the action obtained in [11].

B. Useful formulas

In this section $G(\tau) \equiv G_B(\tau) \equiv G_\tau$, $G(0) \equiv G_0$, $\frac{d}{d\tau}G(\tau) = G'_\tau$, $\frac{d^2}{d\tau^2}G(\tau) = G''_\tau$.

$$\int \frac{d\tau}{2\pi}\frac{d\tau'}{2\pi}\langle \mathrm{e}^{ip\cdot(\xi(\tau)-\xi(\tau'))}\rangle = \int \frac{d\tau}{2\pi}\mathrm{e}^{p^2(G_\tau - G_0)} = \int \frac{d\tau}{2\pi}\ \epsilon^{4p^2}\left[\sin^2\left(\frac{\tau}{2}\right) + \epsilon^2\right]^{-2p^2}$$

[7]This value of $\epsilon_{_R}$ gives the same two-derivative quadratic term as in [11].

$$
\begin{aligned}
&= \frac{1}{\pi}\int_0^1 \frac{dy}{\sqrt{y(1-y)}}\frac{1}{(1+\epsilon^{-2}y)^{2p^2}} = F\left(\frac{1}{2}, 2p^2; 1; -\frac{1}{\epsilon^2}\right) \\
&= \epsilon^{4p^2}\frac{\Gamma(\frac{1}{2}-2p^2)}{\Gamma(\frac{1}{2})\Gamma(1-2p^2)}F\left(2p^2, 2p^2; \frac{1}{2}+2p^2; -\epsilon^2\right) \\
&\quad + \epsilon\frac{\Gamma(-\frac{1}{2}+2p^2)}{\Gamma(\frac{1}{2})\Gamma(2p^2)}F\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}-2p^2; -\epsilon^2\right). \qquad \text{(B.1)}
\end{aligned}
$$

Here, and in what follows, we use the formula

$$
\begin{aligned}
F(a,b;c;z) &= \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)}\frac{1}{(-z)^a}F\left(a, 1-c+a; 1-b+a; \frac{1}{z}\right) + \\
&\quad + \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)}\frac{1}{(-z)^b}F\left(b, 1-c+b; 1-a+b; \frac{1}{z}\right).
\end{aligned}
$$

$$
\int \frac{d\tau}{2\pi}\frac{d\tau'}{2\pi}\langle \mathrm{e}^{ip\cdot(\xi(\tau)-\xi(\tau'))}\dot{\xi}^\mu(\tau)\dot{\xi}^\nu(\tau')\rangle = \left(\delta^{\mu\nu}p^2 - p^\mu p^\nu\right)\int \frac{d\tau}{2\pi}\mathrm{e}^{p^2(G_\tau - G_0)}\left(\frac{d}{d\tau}G(\tau)\right)^2,
$$

$$
\begin{aligned}
\int \frac{d\tau}{2\pi}\mathrm{e}^{p^2(G_\tau-G_0)}\left(\frac{d}{d\tau}G(\tau)\right)^2 &= \int \frac{d\tau}{2\pi}\epsilon^{4p^2}\left[\sin^2\left(\frac{\tau}{2}\right)+\epsilon^2\right]^{-2p^2-2} \times \\
&\quad \times \left(-2\sin\left(\frac{\tau}{2}\right)\cos\left(\frac{\tau}{2}\right)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4}{\pi}\frac{1}{\epsilon^4}\int_0^1 dy\frac{\sqrt{y(1-y)}}{(1+\epsilon^{-2}y)^{2p^2+2}} \\
&= \frac{1}{2}\frac{1}{\epsilon^4}F\left(\frac{3}{2}, 2+2p^2; 3; -\frac{1}{\epsilon^2}\right) \\
&= \epsilon^{4p^2}\frac{2\ \Gamma(-\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)}F\left(2+2p^2, 2p^2; \frac{3}{2}+2p^2; -\epsilon^2\right) + \frac{1}{\epsilon}\frac{2\Gamma(\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(2+2p^2)} \times \\
&\quad \times\ F\left(\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}-2p^2; -\epsilon^2\right) \\
&= \epsilon^{4p^2}\frac{2\ \Gamma(-\frac{1}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)} - \epsilon^{4p^2+2}\frac{4\ (1+p^2)\Gamma(-\frac{3}{2}-2p^2)}{\Gamma(1/2)\Gamma(-2p^2)} + \\
&\quad +\ \frac{1}{\epsilon}\frac{2\ \Gamma(\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(2+2p^2)} - \epsilon\frac{3\ \Gamma(-\frac{1}{2}+2p^2)}{2\Gamma(1/2)\Gamma(2+2p^2)}.
\end{aligned}
$$

$$
\begin{aligned}
&\int \frac{d\tau}{2\pi}\frac{d\tau'}{2\pi}\langle \mathrm{e}^{ip\cdot(\xi(\tau)-\xi(\tau'))}\dot{\xi}^\mu(\tau)\dot{\xi}^\nu(\tau)\dot{\xi}^\rho(\tau')\dot{\xi}^\sigma(\tau')\rangle = \\
&\quad = \int \frac{d\tau}{2\pi}\mathrm{e}^{p^2(G_\tau-G_0)}\Big[\delta^{\mu\nu}\delta^{\rho\sigma}\left(G''_0\right)^2 + \left(\delta^{\mu\rho}\delta^{\nu\sigma}+\delta^{\mu\sigma}\delta^{\nu\rho}\right)\left(G''_\tau\right)^2 + \\
&\qquad + \left(\delta^{\mu\nu}p^\rho p^\sigma + \delta^{\rho\sigma}p^\mu p^\nu\right)G''_0\left(G'_\tau\right)^2 + p^\mu p^\nu p^\rho p^\sigma\left(G'_\tau\right)^4 - \\
&\qquad - \frac{1}{3}p^2\left(\delta^{\mu\rho}p^\nu p^\sigma + \delta^{\mu\sigma}p^\nu p^\rho + \delta^{\nu\rho}p^\mu p^\sigma + \delta^{\nu\sigma}p^\mu p^\rho\right)\left(G'_\tau\right)^4\Big].
\end{aligned}
$$

$$
\int \frac{d\tau}{2\pi}\mathrm{e}^{p^2(G_\tau-G_0)}\left(G''_\tau\right)^2 = \int \frac{d\tau}{2\pi}\epsilon^{4p^2}\left[\sin^2\left(\frac{\tau}{2}\right)+\epsilon^2\right]^{-2p^2}\left(\frac{\sin^2(\tau/2)\left(1+2\epsilon^2\right)-\epsilon^2}{\sin^2(\tau/2)+\epsilon^2}\right)^2
$$

$$
\begin{aligned}
&= \frac{1}{\pi}\frac{1}{\epsilon^8}\int_0^1 \frac{dy}{\sqrt{y(1-y)}}\frac{\left(y\,(1+2\epsilon^2)-\epsilon^2\right)^2}{(1+\epsilon^{-2}y)^{2p^2+4}} \\
&= \frac{1}{32}\frac{1}{\epsilon^8}\left[2^5\epsilon^4 F\left(\frac{1}{2},4+2p^2;1;-\frac{1}{\epsilon^2}\right)-(2+4\epsilon^2)\times\right. \\
&\qquad\left.\times\left(16\epsilon^2 F\left(\frac{3}{2},4+2p^2;2;-\frac{1}{\epsilon^2}\right)-3(2+4\epsilon^2)F\left(\frac{5}{2},4+2p^2;3;-\frac{1}{\epsilon^2}\right)\right)\right] \\
&= \epsilon^{4p^2}\frac{\Gamma(-\frac{3}{2}-2p^2)}{\Gamma(1/2)\Gamma(-1-2p^2)}+\frac{1}{\epsilon^3}\frac{(3+6p^2+4p^4)\Gamma(\frac{3}{2}+2p^2)}{\Gamma(1/2)\Gamma(4+2p^2)}- \\
&\quad -\frac{1}{\epsilon}\frac{4p^2\Gamma(\frac{3}{2}+2p^2)}{\Gamma(1/2)\Gamma(4+2p^2)}\,.
\end{aligned}
$$

$$
\begin{aligned}
\int\frac{d\tau}{2\pi}\mathrm{e}^{p^2(G_\tau-G_0)}\,(G'_\tau)^4 &= \int\frac{d\tau}{2\pi}\epsilon^{4p^2}\left[\sin^2\left(\frac{\tau}{2}\right)+\epsilon^2\right]^{-2p^2}\left(-\frac{2\sin(\tau/2)\cos(\tau/2)}{\sin^2(\tau/2)+\epsilon^2}\right)^4 \\
&= \frac{16}{\pi}\frac{1}{\epsilon^8}\int_0^1 dy\frac{(y(1-y))^{3/2}}{(1+\epsilon^{-2}y)^{2p^2+4}} = \frac{3}{8}\frac{1}{\epsilon^8}F\left(\frac{5}{2},4+2p^2;5;-\frac{1}{\epsilon^2}\right) \\
&= \epsilon^{4p^2}\frac{12\ \Gamma(-\frac{3}{2}-2p^2)}{\Gamma(1/2)\Gamma(1-2p^2)}+\frac{1}{\epsilon^3}\frac{12\ \Gamma(\frac{3}{2}+2p^2)}{\Gamma(1/2)\Gamma(4+2p^2)}-\frac{1}{\epsilon}\frac{45\Gamma(\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(4+2p^2)}\,.
\end{aligned}
$$

The last two terms in (3.17) are given by

$$
\begin{aligned}
B_{\mu\nu}(p)B_{\rho\sigma}(-p)\tilde{K}^{\mu\nu\rho\sigma}(p) &= B_{\mu\nu}(p)B^{\mu\nu}(-p)\frac{2(3+6p^2+4p^4)\Gamma(\frac{3}{2}+2p^2)}{\Gamma(1/2)\Gamma(4+2p^2)} \\
&\quad - p^\mu p^\nu B_{\mu\nu}(p)B^\rho_\rho(-p)\frac{4\ \Gamma(\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(2+2p^2)} \\
&\quad + \frac{12\ \Gamma(\frac{3}{2}+2p^2)}{\Gamma(1/2)\Gamma(4+2p^2)}\left(p^\mu p^\nu B_{\mu\nu}(p)p^\rho p^\sigma B_{\rho\sigma}(-p)-\right. \\
&\qquad\left. -\frac{4}{3}p^2p^\nu p^\sigma B_{\mu\nu}(p)B_{\mu\sigma}(-p)\right)\,, \\
B_{\mu\nu}(p)B_{\rho\sigma}(-p)K^{\mu\nu\rho\sigma}(p) &= -B_{\mu\nu}(p)B^{\mu\nu}(-p)\frac{8p^2\Gamma(\frac{3}{2}+2p^2)}{\Gamma(1/2)\Gamma(4+2p^2)} \\
&\quad + p^\mu p^\nu B_{\mu\nu}(p)B^\rho_\rho(-p)\frac{3\ \Gamma(-\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(2+2p^2)} \\
&\quad - \frac{45\ \Gamma(\frac{1}{2}+2p^2)}{\Gamma(1/2)\Gamma(4+2p^2)}\left(p^\mu p^\nu B_{\mu\nu}(p)p^\rho p^\sigma B_{\rho\sigma}(-p)\ -\right. \\
&\qquad\left. -\frac{4}{3}p^2p^\nu p^\sigma B_{\mu\nu}(p)B_{\mu\sigma}(-p)\right)\,.
\end{aligned}
$$

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